MBS Momentum Testing

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Introduction

- System linear and angular momentums of a joint-connected multibody system (MBS) has two important properties.
  1. Each such momentum is a state of motion corresponding to the generalized coordinates and rates and is independent of the dynamics formalisms. If the dynamics simulation program computes it, then it must be identical to the independently computed momentum state based on the values of the mass, moi's, angular rates, velocities of the bodies and stored momenta in them that are output from the simulation.
  2. They are constant in the inertial space when no external force or torque is exerted on the system. External forces include jet thruster forces, contact forces, and gravity forces on the bodies. For the angular momentum, one can allow gravity forces present as long as they are uniform over all the bodies (i.e. ***g*** acceleration at each cm is identical). This conservation property allows one to verify the correctness of the dynamics formulation that underlies the simulation.
- The following presents the two system momentums of a rigid MBS.

System CM Centric Position Notation

- See Fig 1:

$$\bar{r}_i = \hat{r}_i + R$$

$$\bar{\eta}_i = \hat{\eta}_i + R$$

where $\bar{r}_i =$ inertial position of cm_i

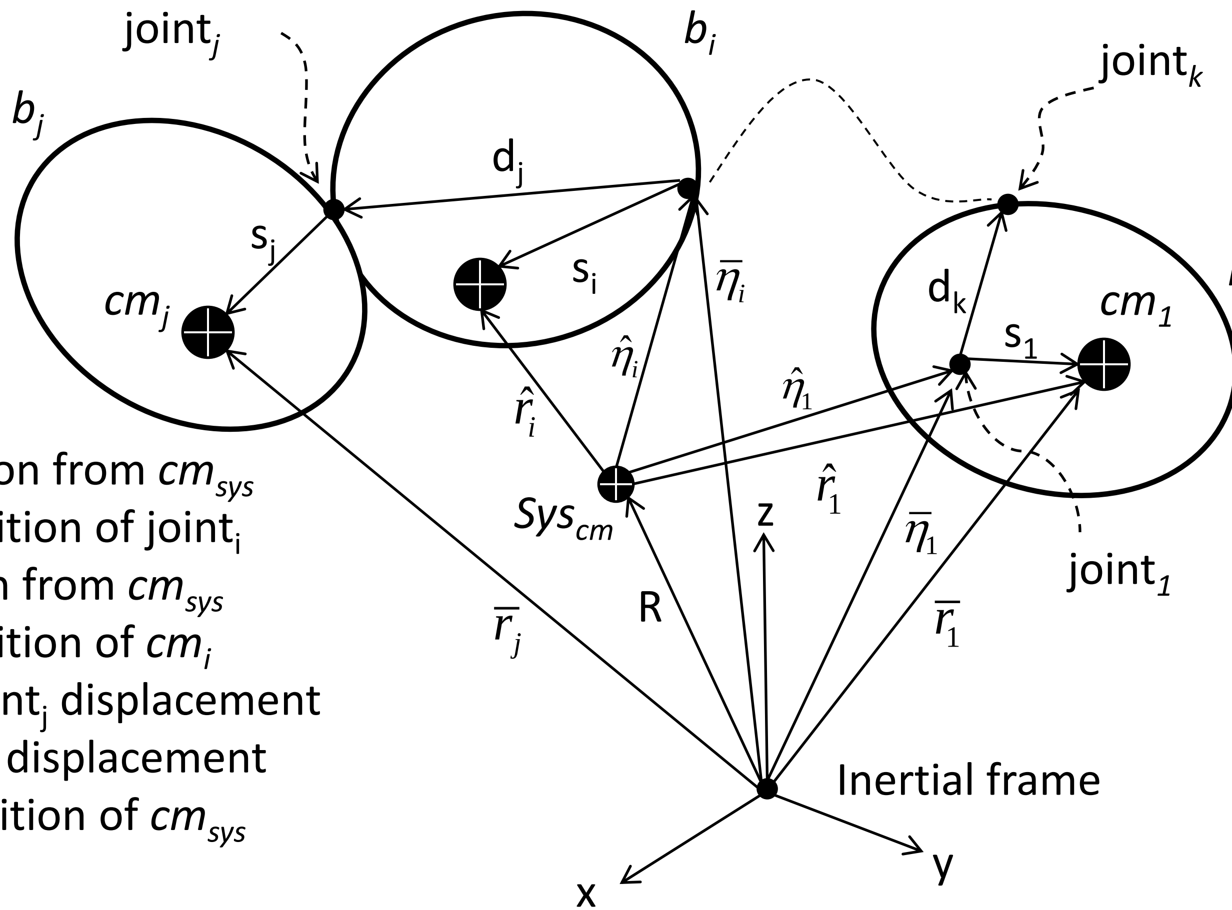
$\bar{\eta}_i =$ inertial position of joint_i

$\hat{r}_i =$ position of cm_i from system cm

$\hat{\eta}_i =$ position of joint_i from system cm

$R =$ inertial position of system cm

Fig. 1 SysCmCentric Notation



b_i $body_i$

$\hat{\eta}_i$ $joint_i$ position from cm_{sys}

$\bar{\eta}_i$ inertial position of $joint_i$

$\hat{r}_i$ cm_i position from cm_{sys}

$\bar{r}_i$ inertial position of cm_i

d_j $joint_{jp}$ to $joint_j$ displacement

s_j $joint_j$ to cm_j displacement

R inertial position of cm_{sys}

b_1 root body

$b_i = b_{jp}$; parent of b_j

Joint 1 Centric Position Notation

- See Fig 2:

$$\bar{r}_i = r_i + \bar{\eta}_1$$

$$\bar{\eta}_i = \eta_i + \bar{\eta}_1$$

$$R = c + \bar{\eta}_1$$

where $\bar{r}_i =$ inertial position of cm_i

$\bar{\eta}_i =$ inertial position of joint_i

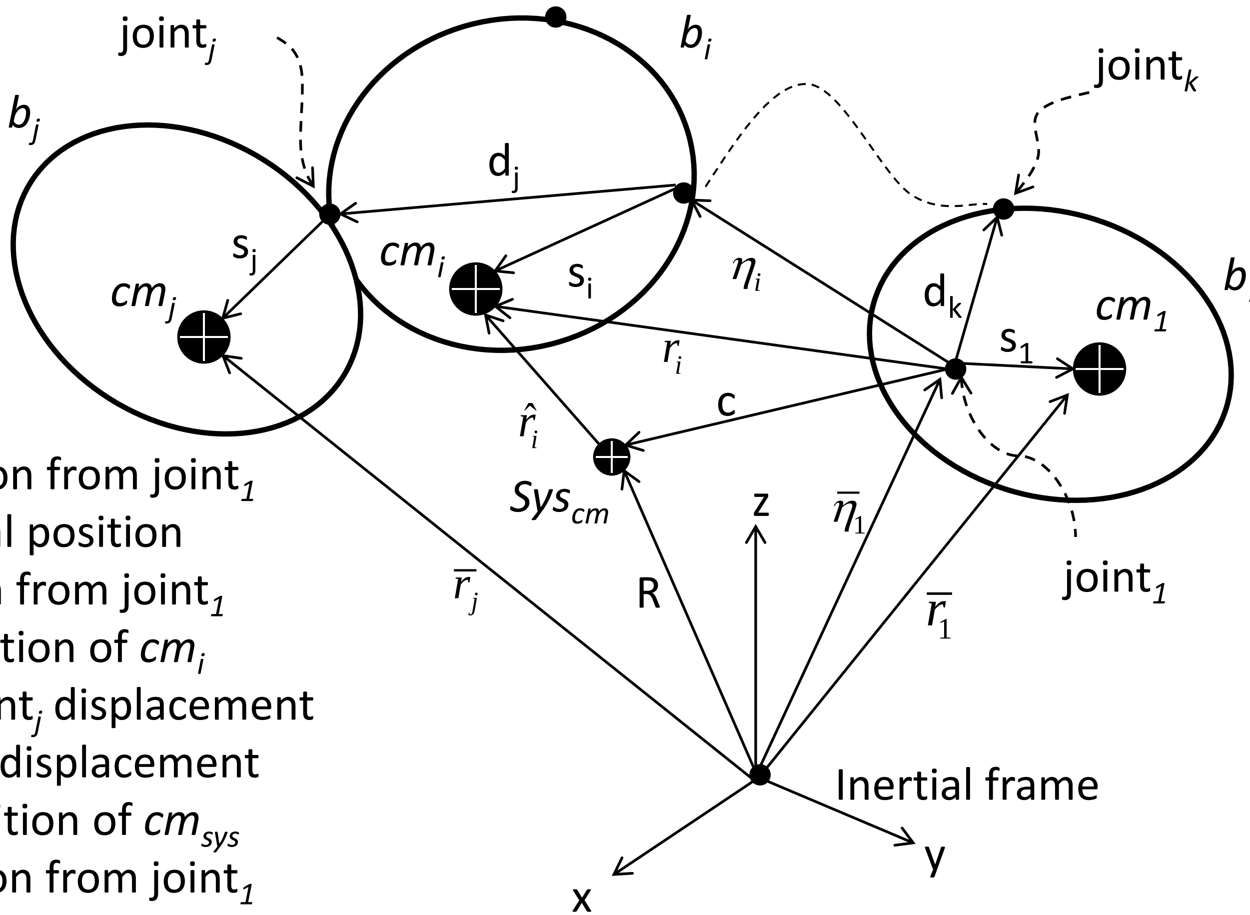
$r_i =$ position of cm_i from $\bar{\eta}_1$

$\eta_i =$ position of joint_i from $\bar{\eta}_1$ with $\eta_1 = 0$

$$c = \frac{\sum_i m_i r_i}{M};\ \text{system cm position from } \eta_1$$

$$M = \sum_i m_i;\ \text{system mass}$$

Fig. 2 Joint 1 Centric Notation



b_i $body_i$

η_i $joint_i$ position from $joint_1$

$\bar{\eta}_i$ $joint_i$ inertial position

r_i cm_i position from $joint_1$

$\bar{r}_i$ inertial position of cm_i

d_j $joint_{jp}$ to $joint_j$ displacement

s_i $joint_i$ to cm_i displacement

R inertial position of cm_{sys}

c cm_{sys} position from $joint_1$

$b_i = b_{jp}$; parent of b_j

2 System Momenta

- Every MBS has two system momentum states, the linear momentum and the angular momentum of all bodies in the system.

- Both momenta are functions of the system generalized coordinates and rates. They can be computed from the total velocities and angular rates of the bodies as shown next.

System Linear Momentum

- The system linear momentum is defined in the inertial frame by

$$P_{sys} = \sum_{i=1:N} m_i \dot{\bar{r}}_i \quad (1)$$

where, m_i = mass of body b_i

$\dot{\bar{r}}_i$ = inertial velocity of cm of b_i

N = number of bodies in the system

System Angular Momentum

- The system angular momentum is defined in the inertial frame by

$$H_{sys} = \sum_{j=1}^{N}(I_j\omega_j + m_j\hat{r}_j \times \dot{\hat{r}}_j) + \sum_{k=1}^{N_w} h_k^w \qquad (2)$$

where $\omega_j =$ angular rate of b_j in inertial coordinates

$I_j =$ moi of b_j, $m_j =$ mass of b_j

$\hat{r}_j =$ position of cm_j from Sys_{cm} in inertial coordinates

$\dot{\hat{r}}_j =$ velocity of cm_j from Sys_{cm} in inertial coordinates

$h_k^w =$ wheel momentum in inertial coordinates

$N_w =$ number of wheels in the system

Derivation of System Angular Momentum Per SysCmCentric Notation, Fig 1

- The following arguments derive the multibody system angular momentum about the system cm in the inertial frame, Eq. (2), using the System CM Centric Notation of Figure 1.

- If a point, C, is the center of mass of a rigid body b_j then the first mass moment about C is zero, i.e.

$$\int_{b_j} l\,dm = 0 \tag{3}$$

where $dm =$ differential mass at l from C

- Angular momentum of a b_j about the system center of mass per Fig. 1 is defined by

$$h_j = \int_{b_j} (l + \hat{r}_j) \times (\dot{l} + \dot{\hat{r}}_j + \dot{R})\,dm \tag{4}$$

where $\hat{r}_j =$ cm of b_j measured from system cm in inertial coordinates

$\dot{\hat{r}}_j = d(\hat{r}_j)/dt$

$\dot{R} =$ inertial velocity of system cm

- Angular momentum of b_j about its own center of mass is

$$\hat{h}_j = \int_{b_j} l \times \dot{l}\, dm = I_j \omega_j \qquad (5)$$

where $\dot{l} = \omega_j \times l$

$\omega_j =$ total angular rate of b_j in inertial coordinates

$I_j = -\int_{b_j} \tilde{l}\,\tilde{l} dm$ in inertial coordinates

$\tilde{l} =$ skew symmetric matrix of l

- Given Eqs. (3,4,5), the angular momentum of b_j about the system center of mass is

$$h_j = I_j \omega_j + m_j \hat{r}_j \times (\dot{\hat{r}}_j + \dot{R}) \qquad (6)$$

where $m_j = \int_{b_j} dm$

- Given Eq. 6, the angular momentum of all *N* bodies in the system about the system center of mass is

$$H_{sys} = \sum_{j=1}^{N} h_j$$

$$= \sum_{j=1}^{N} (I_j \omega_j + m_j \hat{r}_j \times (\dot{\hat{r}}_j + \dot{R})) \qquad (7)$$

- Equation 7 reduces to

$$H_{sys} = \sum_{j=1}^{N} (I_j \omega_j + m_j \hat{r}_j \times \dot{\hat{r}}_j) \qquad (8)$$

since

$\sum_{j=1}^{N} m_j \hat{r}_j = 0$, given that $\hat{r}_j$ is measured from the system cm, see Fig. 1

- For systems that have N_w spinning wheels, Eq. 8 becomes

$$H_{sys} = \sum_{j=1}^{N} (I_j \omega_j + m_j \hat{r}_j \times \dot{\hat{r}}_j) + \sum_{k=1}^{N_w} h_k^w \qquad (9)$$

where $h_k^w = g_k^w I_k^w \dot{\theta}_k$; wheel momentum

$g_k^w =$ wheel spin axis in inertial coordinates

$I_k^w =$ wheel spin axis inertia

$\dot{\theta}_k =$ wheel spin speed

- Equation (9) is Eq. (2). QED

Compute H_{sys}: Procedure 1

- Equation. 9 is a function of $\left\{m_j, I_j, \omega_j, \hat{r}_j, \dot{\hat{r}}_j, h_k^w\right\}$ for $j = 1:N, k = 1:N_w$ in System CM centric notation (Fig. 1).
- H_{sys} is independent of $\dot{R}$, the total velocity of the system cm
- If the simulation output $\left\{I_j, \omega_j, \hat{r}_j, \dot{\hat{r}}_j, h_k^w\right\}$ for $j = 1:N, k = 1:N_w$ in inertial coordinates, then use that set for Eq. 9 to computes H_{sys} in inertial coordinates
- Body mass set $\left\{m_j\right\}_{j=1:N}$ can be either from the simulation or user supplied

Other Procedures to Compute H_{sys}

- Three other procedures to compute H_{sys} are given here when the key kinematics parameters from simulation output are different from those required by Eq. 9 for the H_{sys} computation.

Compute H_{sys}: Procedure 2

- If the simulation output $\left\{\omega_j, \bar{r}_j, \dot{\bar{r}}_j, h_k^w, I_j\right\}$ in inertial coordinates (Fig. 1), then compute H_{sys} by the following procedure:
    1. compute $M = \sum_i m_i$; $R = \dfrac{\sum_{j=1}^{N} m_j \bar{r}_j}{M}$; $\dot{R} = \dfrac{\sum_{j=1}^{N} m_j \dot{\bar{r}}_j}{M}$
    2. compute $\hat{r}_j = \bar{r}_j - R,\ \dot{\hat{r}}_j = \dot{\bar{r}}_j - \dot{R}$ for $j = 1:N$
    3. use $\left\{m_j, I_j, \omega_j, \hat{r}_j, \dot{\hat{r}}_j, h_k^w\right\}_{j=1:N,k=1:N_w}$ for Eq. 9 to compute H_{sys}

- Body mass set $\left\{m_j\right\}_{j=1:N}$ can be either simulation or user supplied

Compute H_{sys}: Procedure 3

- If the simulation output $\left\{\omega_j^1, \hat{r}_j^1, \dot{\hat{r}}_j^1, [h_k^w]^1, I_j^1, C_1\right\}$ in b_1 coordinates in System CM centric notation (Fig. 1), then compute H_{sys} by the following procedure
    1. compute $\hat{r}_j = C_1\hat{r}_j^1$, $\dot{\hat{r}}_j = C_1\dot{\hat{r}}_j^1$, $I_j = C_1 I_j^1 C_1^T$, $\omega_j = C_1\omega_j^1$ for $j = 1:N$
    2. compute $h_k^w = C_1[h_k^w]^1$ for $k = 1:N_w$
    3. use $\left\{m_j, I_j, \omega_j, \hat{r}_j, \dot{\hat{r}}_j, h_k^w\right\}_{j=1:N,k=1:N_w}$ for Eq. 9 to compute H_{sys}

Compute H_{sys}: Procedure 4

- If the simulation output $\left\{\omega_j^1, r_j^1, \dot{r}_j^1, [h_k^w]^1, I_j^1, C_1\right\}$ in b_1 coordinates in Joint 1 Centric notation (Fig. 2), then compute H_{sys} by the following procedure
    1. compute $M = \sum_i m_i$; $c^1 = \left(\sum_{j=1}^{N} m_j r_j^1\right) / M$; $\dot{c}^1 = \left(\sum_{j=1}^{N} m_j \dot{r}_j^1\right) / M$
    2. compute $\hat{r}_j = C_1(r_j^1 - c^1)$, $\dot{\hat{r}}_j = C_1(\dot{r}_j^1 - \dot{c}^1)$, $I_j = C_1 I_j^1 C_1^T$, $\omega_j = C_1 \omega_j^1$ for $j = 1:N$
    3. compute $h_k^w = C_1[h_k^w]^1$ for $k = 1:N_w$
    4. use $\left\{m_j, I_j, \omega_j, \hat{r}_j, \dot{\hat{r}}_j, h_k^w\right\}_{j=1:N, k=1:N_w}$ for Eq. 9 to compute H_{sys}

Summary

- Momentum testing means comparison of independently computed P_{sys} (Eq. 1) and H_{sys} (Eq. 9) versus those produced by the dynamics simulation program.

- Between P_{sys} and H_{sys} consistency testing, the H_{sys} test is better because it covers more information than the P_{sys} test, i.e. H_{sys} calculation includes positions, angular velocities and the stored momenta that are not in P_{sys}.

- Post-sim Calculation of P_{sys} and H_{sys} of a rigid multibody system by Eqs. (1) or (9) requires no knowledge of the system joint-topology and it is valid for any dynamics formulation method used by the simulation.

- Procedures to compute H_{sys} of a rigid multibody system for 4 types of output from the simulation have been presented using Eq. (9).

- H_{sys} testing (post-sim) serves to:
  - Identify differences in the momentum state between the one computed by the simulation and that of independently computed based on the position and rate data output from the simulation to check for mass property or kinematics discrepancies
  - Verify momentum conservation of the multibody dynamics simulation

- Passing the H_{sys} test means that the dynamics principle underlying the simulation and its implementation are credible and the simulation result is useful.

- H_{sys} test must be part of any multibody dynamics simulation validation process.