MBS Energy Testing

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Introduction

- Kinetic (KE) and potential (PE) energy of a joint-connected multibody system (MBS) are useful conserved quantities to evaluate in validating simulations under controlled conditions.
  1. If KE and PE are not explicitly time dependent, and the only forces in the system are conservative ones then the total energy (KE+PE) of the system is constant.
  2. If KE and PE are not explicitly time dependent, and no internal nor external forces are present in the system, then KE of the system is constant.

- MBS energy testing means
  1. If the KE and PE computed internally by the simulation are identical to those independently computed off-line, then the mass property used by the simulation is implemented correctly. The independent computation is done using the velocities and angular rate vectors output by the simulation.
  2. If the KE and PE are conserved for the conditions stated above, then the MBD formulation and implementation underlying the simulation is properly done and the solutions under forced conditions are credible. At that point the simulation is ready for control performance evaluation.

Position and Velocity Representations

- Two practical MBS notations for positions and velocities are:
  - System CM centric notation
  - Joint(1) centric notation

- These are defined in the following charts.

System CM Centric Position Equations

- The inertial position of a body and that of its joint are defined based on the system cm position (see Fig. 1):

$$\bar{r}_i = \hat{r}_i + R$$

$$\bar{\eta}_i = \hat{\eta}_i + R$$

where $\bar{r}_i =$ inertial position of cm_i

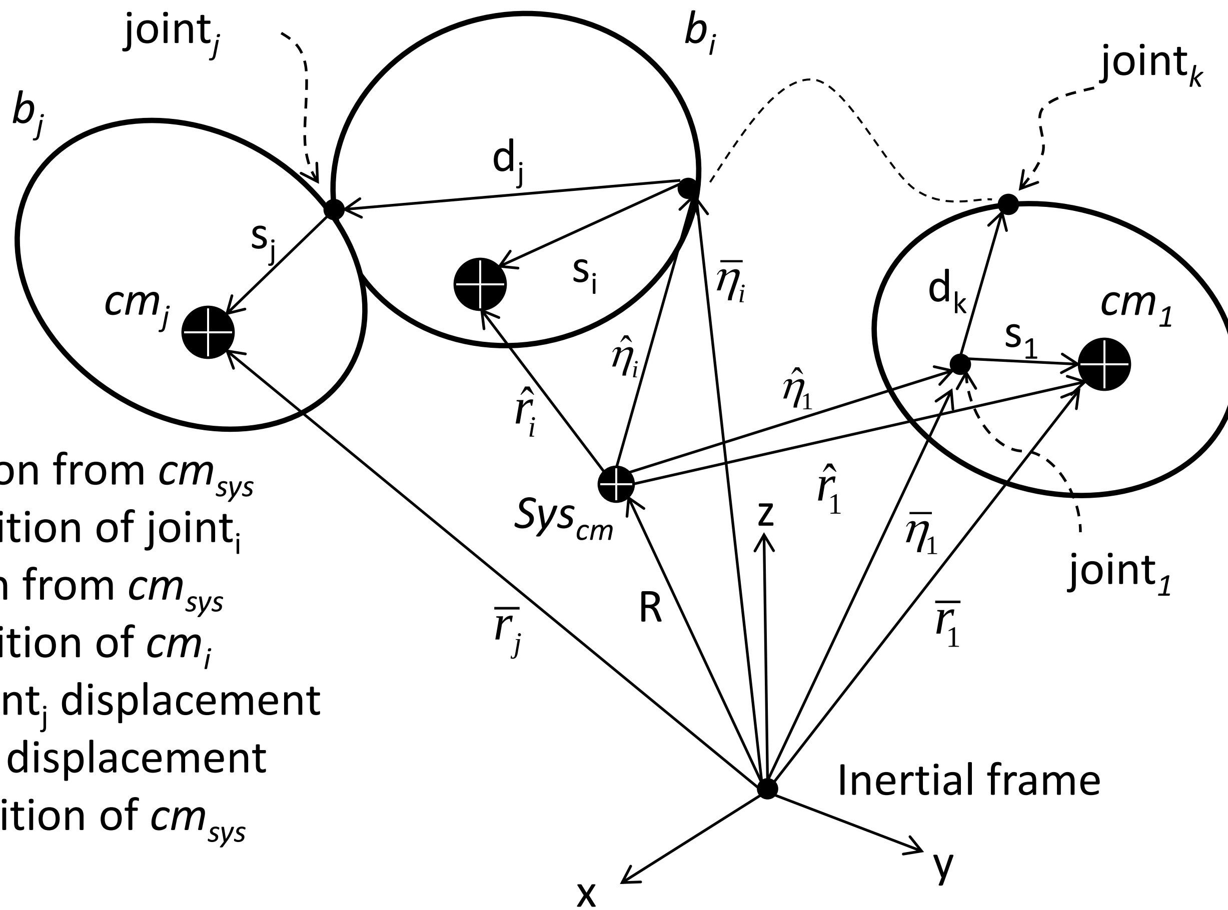
$\bar{\eta}_i =$ inertial position of joint_i

$\hat{r}_i =$ position of cm_i from system cm

$\hat{\eta}_i =$ position of joint_i from system cm

$R =$ inertial position of system cm

Fig. 1 SysCmCentric Notation



b_i body$_i$

$\hat{\eta}_i$ joint$_i$ position from cm_{sys}

$\bar{\eta}_i$ inertial position of joint$_i$

$\hat{r}_i$ cm_i position from cm_{sys}

$\bar{r}_i$ inertial position of cm_i

d_j joint$_{jp}$ to joint$_j$ displacement

s_j joint$_j$ to cm_j displacement

R inertial position of cm_{sys}

b_1 root body

$b_i = b_{jp}$; parent of b_j

Joint (1) Centric Position Equations

- The inertial position of a body and that of its joint are defined based on the joint(1) inertial position (See Fig 2):

$$\bar{r}_i = r_i + \bar{\eta}_1$$

$$\bar{\eta}_i = \eta_i + \bar{\eta}_1$$

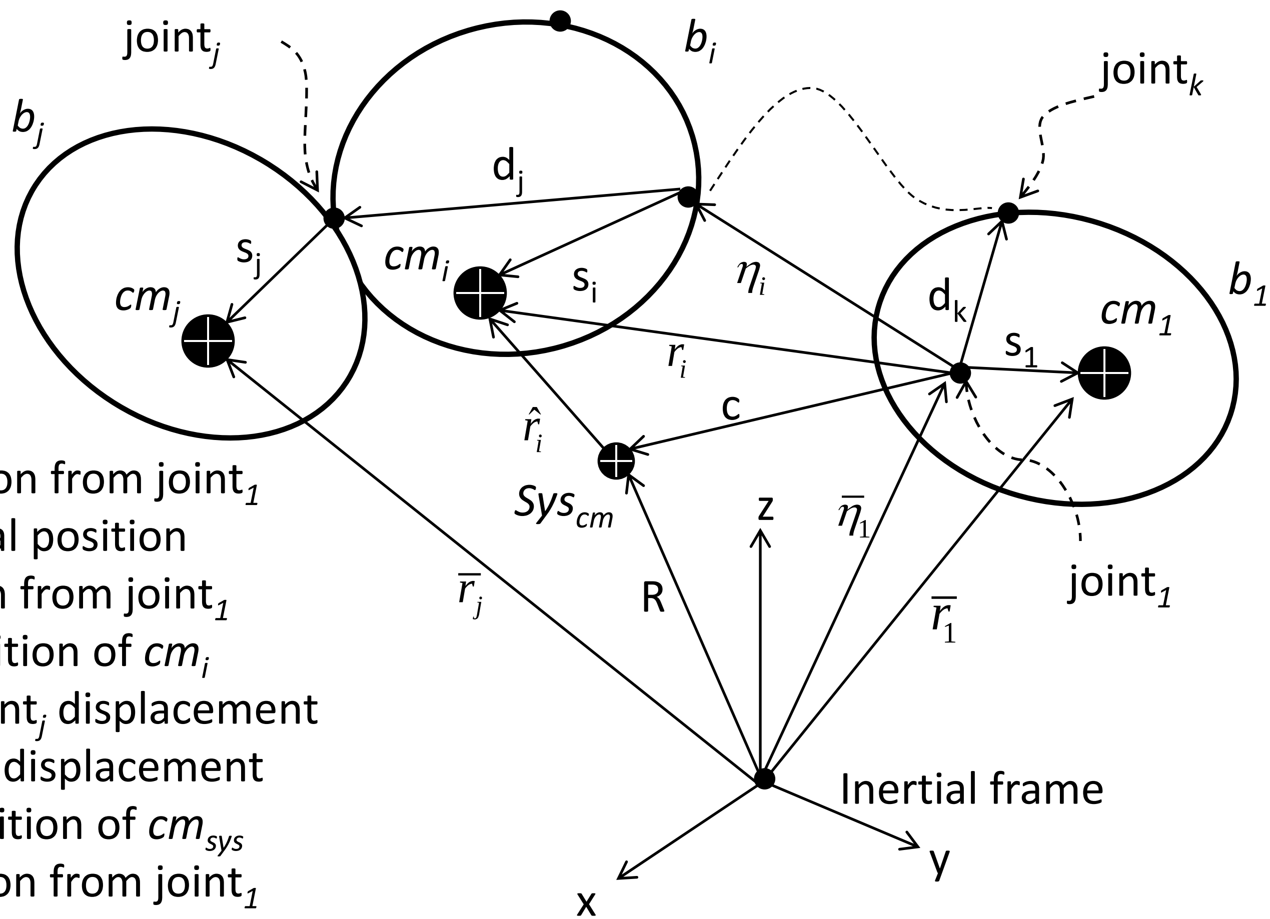
where $\bar{r}_i = \text{inertial position of cm}_i$

$\bar{\eta}_i = \text{inertial position of joint}_i$

$r_i = \text{position of cm}_i \text{ from } \bar{\eta}_1$

$\eta_i = \text{position of joint}_i \text{ from } \bar{\eta}_1 \text{ with } \eta_1 = 0$

Fig. 2 Joint (1) Centric Notation



b_i body$_i$

η_i joint$_i$ position from joint$_1$

$\bar{\eta}_i$ joint$_i$ inertial position

r_i cm_i position from joint$_1$

$\bar{r}_i$ inertial position of cm_i

d_j joint$_{jp}$ to joint$_j$ displacement

s_i joint$_i$ to cm_i displacement

R inertial position of cm_{sys}

c cm_{sys} position from joint$_1$

$b_i = b_{jp}$; parent of b_j

System KE

- Kinetic energy of an N-body system that has N_w wheels is defined by

$$KE = \frac{1}{2}\left(\sum_{i=1}^{N} (\omega_i^T I_i \omega_i + m_i \dot{\bar{r}}_i^T \dot{\bar{r}}_i) + \sum_{k=1}^{N_w} \frac{h_k^{w,T} h_k^w}{\hat{I}_k^w} \right) \qquad (1)$$

where ω_i = angular rate of b_i in inertial coordinates

I_i = moi of b_i, m_i = mass of b_i

$\bar{r}_i$ = inertial position of cm_i

$\dot{\bar{r}}_i$ = inertial velocity of cm_i

h_k^w = wheel momentum in inertial coordinates

$\hat{I}_k^w$ = wheel spin axis moment of inertia

System PE

- Potential energy of an N-body system with springs at the joints and placed in a constant gravity field is defined by

$$PE = \frac{1}{2}\sum_{i=1}^{N} k_i \theta_i^2 - \sum_{i=1}^{N} m_i \vec{g}^T \overline{r}_i \qquad (2)$$

where θ_i = joint angle

k_i = joint spring stiffness

m_i = mass of b_j

$\overline{r}_i$ = inertial position of cm_i

$\vec{g}$ = gravitational acceleration in inertial coordinates

Compute System Energy: Procedure 1

- Let the simulation output $\left\{ m_i, I_i, \omega_i, \overline{r}_i, \dot{\overline{r}}_i, h_k^w \right\}$ for $i = 1 : N, k = 1 : N_w$ with all vector and diadic quantities in inertial coordinates.
- Body mass set $\left\{ m_j \right\}_{j=1:N}$ can be either from the simulation or user supplied
- Use the given output to compute *KE* and *PE* by Eqs. (1) and (2).

Other Procedures to Compute *KE* and *PE*

- Three other procedures to compute *System energy* are given here when the key kinematics parameters from simulation output are different from those required by Eqs. (1) and (2) for the *KE* and *PE* computations.

Compute System Energy: Procedure 2

- If the simulation output $\left\{\omega_j, \hat{r}_j, \dot{\hat{r}}_j, h_k^w, I_j, R, \dot{R}\right\}$ with all vectors and diadic quantities given in inertial coordinates (see Fig. 1), then compute the energies by the following procedure:

  1. compute $\overline{r}_i = \hat{r}_i + R,\ \dot{\overline{r}}_i = \dot{\hat{r}}_i + \dot{R}$ for $i = 1:N$
  2. use $\left\{m_i, I_i, \omega_i, \overline{r}_i, \dot{\overline{r}}_i, h_k^w\right\}_{i=1:N,k=1:N_w}$ for Eqs. (1) and (2) to compute KE and PE

- Body mass set $\left\{m_j\right\}_{j=1:N}$ can be either simulation or user supplied

Compute System Energy: Procedure 3

- If the simulation output $\left\{\omega_i^1, \hat{r}_i^1, \dot{\hat{r}}_i^1, [h_k^w]^1, I_i^1, R, \dot{R}, C_1\right\}$ with all vectors and diadic quantities given in b_1 coordinates under the System CM centric notation (see Fig. 1), with $\left(R, \dot{R}\right)$ in inertial coordinates, then the energies are computed as follows

1. compute $I_i = C_1 I_i^1 C_1^T$, $\omega_i = C_1 \omega_i^1$ for $i = 1:N$

   where $C_1 =$ coordinate transformation matrix from b_1 to inertial coords.

2. compute $\overline{r}_i = C_1 \hat{r}_i^1 + R$, $\dot{\overline{r}}_i = C_1 \dot{\hat{r}}_i^1 + \dot{R}$ for $i = 1:N$

3. use $\left\{m_i, I_i, \omega_i, \overline{r}_i, \dot{\overline{r}}_i, h_k^w\right\}_{i=1:N,\, k=1:N_w}$ for Eqs. (1) and (2) to compute KE and PE

- The stored wheel momenta $\left\{[h_k^w]^1\right\}_{k=1}^{N_w}$ need not be put in inertial coordinates, since it's the sum of their dot products that's needed for *KE*.

Compute System Energy: Procedure 4

- If the simulation output $\left\{\omega_i, r_i, \dot{r}_i, h_k^w, I_i, \eta_1, \dot{\eta}_1\right\}$ for $i=1:N, k=1:N_w$ with all vectors and diadic quatities given in inertial coordinates under the Joint(1) Centric notation (see Fig. 2), then the system energies can be computed as follows
    1. compute $\overline{r}_i = r_i + \eta_1, \dot{\overline{r}}_i = \dot{r}_i + \dot{\eta}_1$ for $i=1:N$
    2. use $\left\{m_j, I_j, \omega_j, \overline{r}_i, \dot{\overline{r}}_i, h_k^w\right\}_{j=1:N, k=1:N_w}$ for Eqs. (1) and (2) for KE and PE

Summary

- Energy testing means:
  1. Independently computed *KE* and *PE* by Eqs. (1) and (2) must be identical to those done by the simulation
  2. The following energy response must be observed for the following force conditions.
     a) (KE + PE)=constant if no net external forces/torque are imposed on the MBS; spring forces are allowed if the stored spring energy is included in PE (see Eq. 2)
     b) KE=constant if no net external forces/torque are imposed on the MBS and no joint forces are present in the MBS

- Post-sim Calculation of *KE* and *PE* of a rigid multibody system by Eqs. (1) and (2) requires no knowledge of the system joint-topology and it is valid for any dynamics formulation method used by the simulation.

- Procedures to compute *KE* and *PE* of a rigid multibody system for 4 types of output from the simulation were presented using Eqs. (1) and (2).

- *System energy tests* serve to:
  - check the correctness of mass property and the kinematics eqs. used
  - Verify energy conservation capability of the MBD formulation underlying the dynamics simulation

- Passing the energy tests under the stated conditions means that the dynamics principle underlying the simulation and its implementation are credible and that one can proceed to examine the control performance of the simulation.

- Also check System Momentum Tests

- System energy tests must be part of any multibody dynamics simulation validation process.