MBS Angular Momentum Testing

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Introduction

- The total angular momentum of a multibody system (MBS) or a system of interconnected bodies has two important properties.

    1. It's a state of motion corresponding to the generalized coordinates and rates and is independent of the dynamics formalisms. If the dynamics simulation program computes it, then it must be identical to the independently computed momentum state based on the values of the moi's, angular rates, positions, velocities of the bodies and stored momenta in them that are output by the simulation.

    2. It is constant in the inertial space when no external torque is exerted on the system. External forces include jet thruster forces, contact forces, and gravity forces on the bodies. In the case gravity forces are present, the gravitational acceleration must be uniform over all the bodies (i.e. ***g*** acceleration at each cm is identical). This property allows one to verify the correctness of the dynamics formulation that underlies the simulation.

- The following presents the derivation of the system angular momentum of a rigid MBS.

System CM Centric Position Notation

- See Fig 1:

$$\bar{r}_i = \hat{r}_i + R$$

$$\bar{\eta}_i = \hat{\eta}_i + R$$

where $\bar{r}_i = \text{inertial position of } \text{cm}_i$

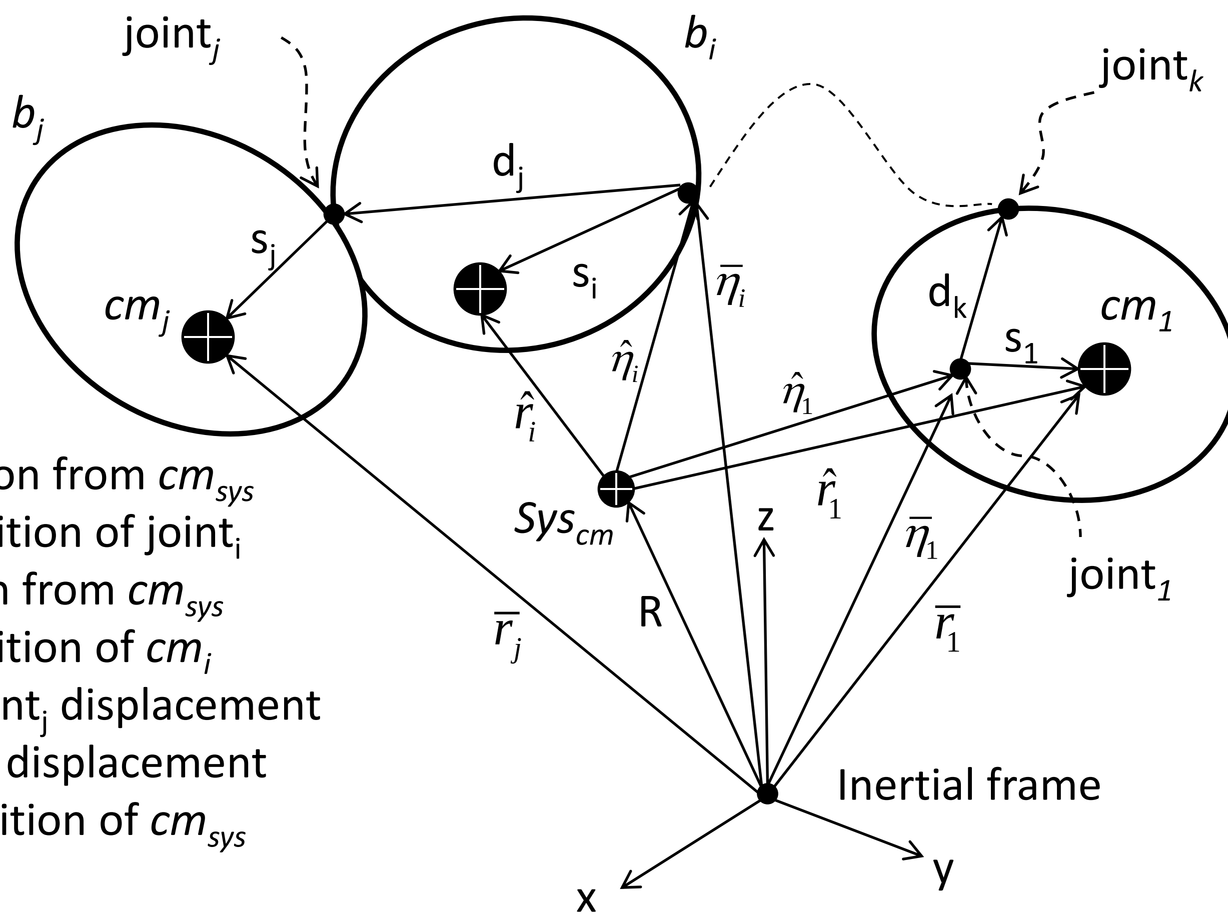
$\bar{\eta}_i = \text{inertial position of } \text{joint}_i$

$\hat{r}_i = \text{position of } \text{cm}_i \text{ from system cm}$

$\hat{\eta}_i = \text{position of } \text{joint}_i \text{ from system cm}$

$R = \text{inertial position of system cm}$

Fig. 1 SysCmCentric Notation



b_i body$_i$

$\hat{\eta}_i$ joint$_i$ position from cm_{sys}

$\bar{\eta}_i$ inertial position of joint$_i$

$\hat{r}_i$ cm_i position from cm_{sys}

$\bar{r}_i$ inertial position of cm_i

d_j joint$_{jp}$ to joint$_j$ displacement

s_j joint$_j$ to cm_j displacement

R inertial position of cm_{sys}

b_1 root body

$b_i = b_{jp}$; parent of b_j

Joint 1 Centric Position Notation

- See Fig 2:

$$\bar{r}_i = r_i + \bar{\eta}_1$$

$$\bar{\eta}_i = \eta_i + \bar{\eta}_1$$

$$R = c + \bar{\eta}_1$$

where $\bar{r}_i = \text{inertial position of cm}_i$

$$\bar{\eta}_i = \text{inertial position of joint}_i$$

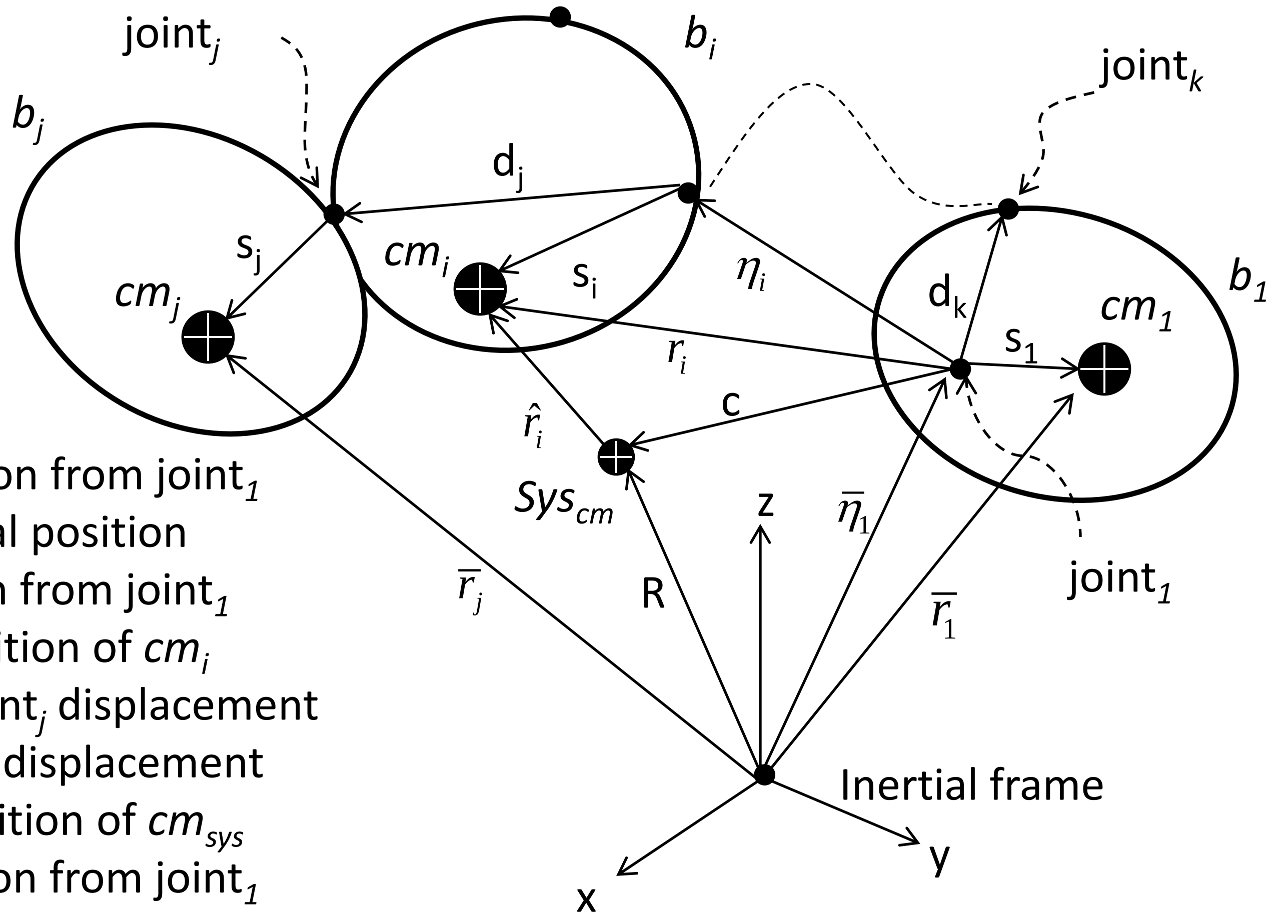
$$r_i = \text{position of cm}_i \text{ from } \bar{\eta}_1$$

$$\eta_i = \text{position of joint}_i \text{ from } \bar{\eta}_1 \text{ with } \eta_1 = 0$$

$$c = \frac{\sum_i m_i r_i}{M}; \text{ system cm position from } \eta_1$$

$$M = \sum_i m_i; \text{ system mass}$$

Fig. 2 Joint 1 Centric Notation



b_i body$_i$

η_i joint$_i$ position from joint$_1$

$\bar{\eta}_i$ joint$_i$ inertial position

r_i cm_i position from joint$_1$

$\bar{r}_i$ inertial position of cm_i

d_j joint$_{jp}$ to joint$_j$ displacement

s_i joint$_i$ to cm_i displacement

R inertial position of cm_{sys}

c cm_{sys} position from joint$_1$

b_i=b_{jp}; parent of b_j

System Angular Momentum Per SysCmCentric Notation, Fig 1

- The following arguments leading to Eq. (7) are the derivation of the multibody system angular momentum about the system cm in inertial frame using the System CM Centric Notation of Figure 1.

- If a point, C, is the center of mass of a rigid body b_j then the first mass moment about C is zero, i.e.

$$\int_{b_j} l\,dm = 0 \qquad (1)$$

where dm = differential mass at l from C

- Angular momentum of a body b_j about the system center of mass per Fig. 1 is defined by

$$h_j = \int_{b_j} (l + \hat{r}_j) \times (\dot{l} + \dot{\hat{r}}_j + \dot{R})\,dm \qquad (2)$$

where $\hat{r}_j =$ cm of b_j measured from system cm in inertial coordinates

$\dot{\hat{r}}_j = d(\hat{r}_j)/dt$

$\dot{R} =$ inertial velocity of system cm

- Angular momentum of body b_j about its own center of mass is

$$\hat{h}_j = \int_{b_j} l \times \dot{l}\, dm = I_j \omega_j \qquad (3)$$

where $\dot{l} = \omega_j \times l$

ω_j = total angular rate of b_j in inertial coordinates

$I_j = -\int_{b_j} \tilde{l}\,\tilde{l}\,dm$ in inertial coordinates

$\tilde{l}$ = skew symmetric matrix of l

- Given Eqs. 1 to 3, the angular momentum of body b_j about the system center of mass is

$$h_j = I_j \omega_j + m_j \hat{r}_j \times (\dot{\hat{r}}_j + \dot{R}) \qquad (4)$$

where $m_j = \int_{b_j} dm$

- Given Eq. 4, the angular momentum of all bodies (*N*) in the system about the system center of mass is

$$H_{sys} = \sum_{j=1}^{N} h_j$$

$$= \sum_{j=1}^{N} (I_j \omega_j + m_j \hat{r}_j \times (\dot{\hat{r}}_j + \dot{R})) \qquad (5)$$

- Equation 5 reduces to

$$H_{sys} = \sum_{j=1}^{N} (I_j \omega_j + m_j \hat{r}_j \times \dot{\hat{r}}_j) \qquad (6)$$

since

$\sum_{j=1}^{N} m_j \hat{r}_j = 0$, given that $\hat{r}_j$ is measured from system cm, see Fig. 1

- For those systems that have N_w spinning wheels, Eq. 6 becomes

$$H_{sys} = \sum_{j=1}^{N} (I_j \omega_j + m_j \hat{r}_j \times \dot{\hat{r}}_j) + \sum_{k=1}^{N_w} h_k^w \qquad (7)$$

where $h_k^w = g_k^w I_k^w \dot{\theta}_k$; wheel momentum

$g_k^w =$ wheel spin axis in inertial coordinates

$I_k^w =$ wheel spin axis inertia

$\dot{\theta}_k =$ wheel spin speed

Compute H_{sys}: Procedure 1

- Equation. 7 is a function of $\left\{m_j, I_j, \omega_j, \hat{r}_j, \dot{\hat{r}}_j, h_k^w\right\}$ for $j = 1:N, k = 1:N_w$ in System CM centric notation (Fig. 1).

- H_{sys} is independent of $\dot{R}$, the total velocity of the system cm

- If the simulation output $\left\{I_j, \omega_j, \hat{r}_j, \dot{\hat{r}}_j, h_k^w\right\}_{j=1:N, k=1:N_w}$ in inertial coordinates, then input that set to Eq. 7 to computes H_{sys} in inertial coordinates

- Body mass $\left\{m_j\right\}_{j=1:N}$ can be either simulation or user supplied

Other Procedures to Compute H_{sys}

- Three other procedures to compute H_{sys} are given here when the key kinematics parameters from simulation output are different from those required by Eq. 7 for the H_{sys} computation.

Compute H_{sys}: Procedure 2

- If the simulation output $\left\{\omega_j, \overline{r}_j, \dot{\overline{r}}_j, h_k^w, I_j\right\}$ in inertial coordinates (Fig. 1), then compute H_{sys} by the following procedure:

1. compute $M = \sum_i m_i$; $R = \dfrac{\sum_{j=1}^{N} m_j \overline{r}_j}{M}$; $\dot{R} = \dfrac{\sum_{j=1}^{N} m_j \dot{\overline{r}}_j}{M}$
2. compute $\hat{r}_j = \overline{r}_j - R,\ \dot{\hat{r}}_j = \dot{\overline{r}}_j - \dot{R}$ for $j = 1:N$
3. input $\left\{m_j, I_j, \omega_j, \hat{r}_j, \dot{\hat{r}}_j, h_k^w\right\}_{j=1:N, k=1:N_w}$ to Eq. 7 to compute H_{sys}

- Body mass $\left\{m_j\right\}_{j=1:N}$ can be either simulation or user supplied

Compute H_{sys}: Procedure 3

- If the simulation output $\left\{\omega_j^1, \hat{r}_j^1, \dot{\hat{r}}_j^1, [h_k^w]^1, I_j^1, C_1\right\}$ in b_1 coordinates in System CM centric notation (Fig. 1), then compute H_{sys} by the following procedure

  1. compute $\hat{r}_j = C_1\hat{r}_j^1$, $\dot{\hat{r}}_j = C_1\dot{\hat{r}}_j^1$, $I_j = C_1 I_j^1 C_1^T$, $\omega_j = C_1\omega_j^1$ for $j = 1:N$
  2. compute $h_k^w = C_1[h_k^w]^1$ for $k = 1:N_w$
  3. input $\left\{m_j, I_j, \omega_j, \hat{r}_j, \dot{\hat{r}}_j, h_k^w\right\}_{j=1:N, k=1:N_w}$ to Eq. 7 to compute H_{sys}

Compute H_{sys}: Procedure 4

- If the simulation output $\left\{\omega_j^1, r_j^1, \dot{r}_j^1, [h_k^w]^1, I_j^1, C_1\right\}$ in b_1 coordinates in Joint 1 Centric notation (Fig. 2), then compute H_{sys} by the following procedure

1. compute $M = \sum_i m_i \;;\; c^1 = \left(\sum_{j=1}^{N} m_j r_j^1\right)/M \;;\; \dot{c}^1 = \left(\sum_{j=1}^{N} m_j \dot{r}_j^1\right)/M$
2. compute $\hat{r}_j = C_1(r_j^1 - c^1),\ \dot{\hat{r}}_j = C_1(\dot{r}_j^1 - \dot{c}^1),\ I_j = C_1 I_j^1 C_1^T,\ \omega_j = C_1 \omega_j^1$ for $j = 1:N$
3. compute $h_k^w = C_1 [h_k^w]^1$ for $k = 1:N_w$
4. input $\left\{m_j, I_j, \omega_j, \hat{r}_j, \dot{\hat{r}}_j, h_k^w\right\}_{j=1:N,\, k=1:N_w}$ to Eq. 7 to compute H_{sys}

Summary

- Procedures to compute H_{sys} of a rigid multibody system for 4 types of output from the simulation have been presented using Eqs. (6,7).

- H_{sys} of a rigid multibody system can be computed from the velocity vectors of member bodies of the system and stored momentum of spinning wheels in the system, i.e. Eqs. (6,7). This calculation does not require knowledge of the system joint-topology.

- H_{sys} testing (post-sim) serves to:
  - Identify differences in the momentum state between the one computed by the simulation and that of independently computed based on the position and rate data output from the simulation to check for mass property or kinematics discrepancies
  - Verify momentum conservation of the multibody dynamics simulation